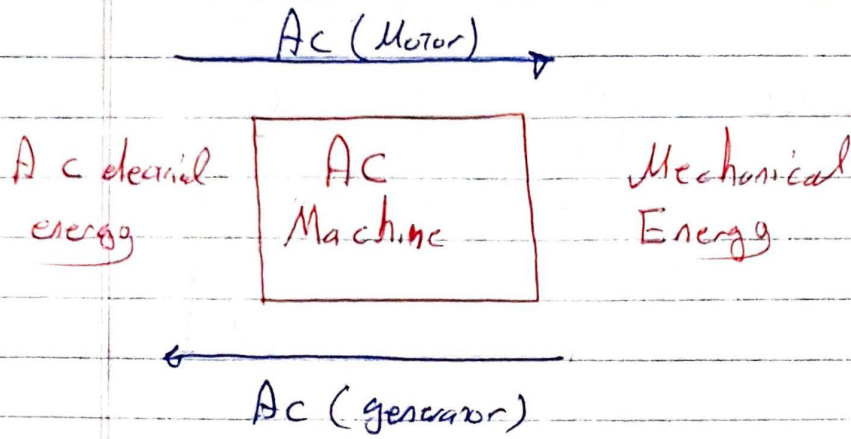


CH3 Fundamentals of AC Machinery.

9+1+10+8
+5+



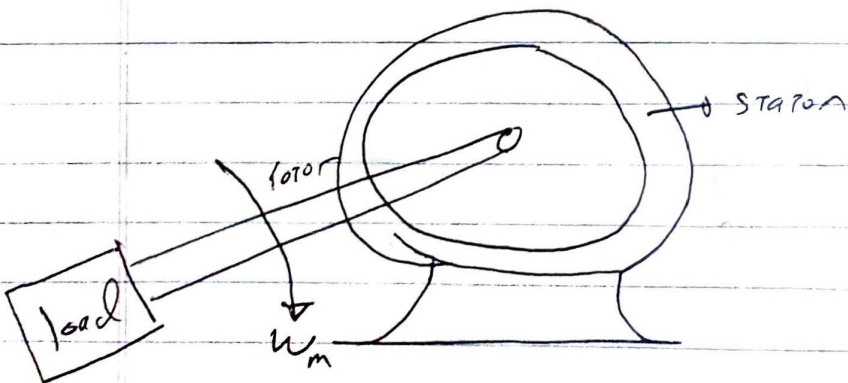
⊗ Types of AC Machines :-

- ① Synchronous Machines :- They are generator or Motors whose magnetic field current is supplied by separate DC source.
- ② Inductions Machines :- They are Motors or generator whose magnetic field current is supplied by Magnetic Induction (Transformer Action)

⊗ CONSTRUCTION of AC Machines :-

The AC machine consists of two main parts :-

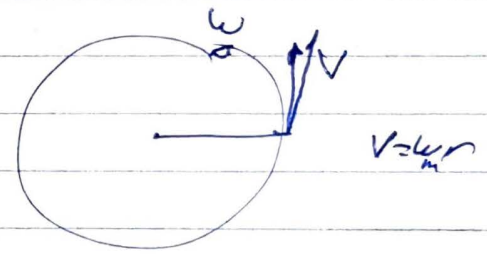
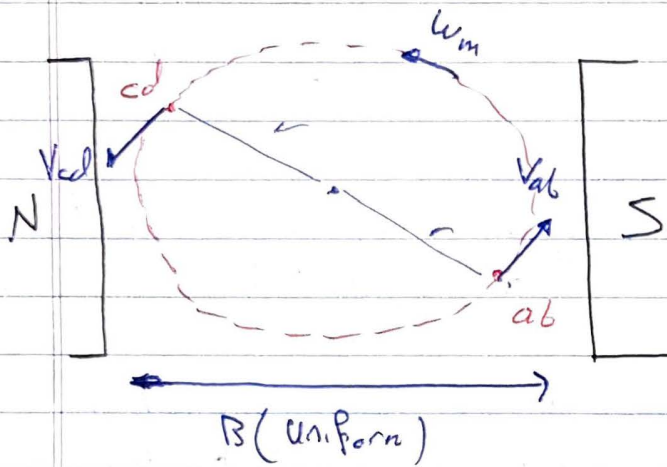
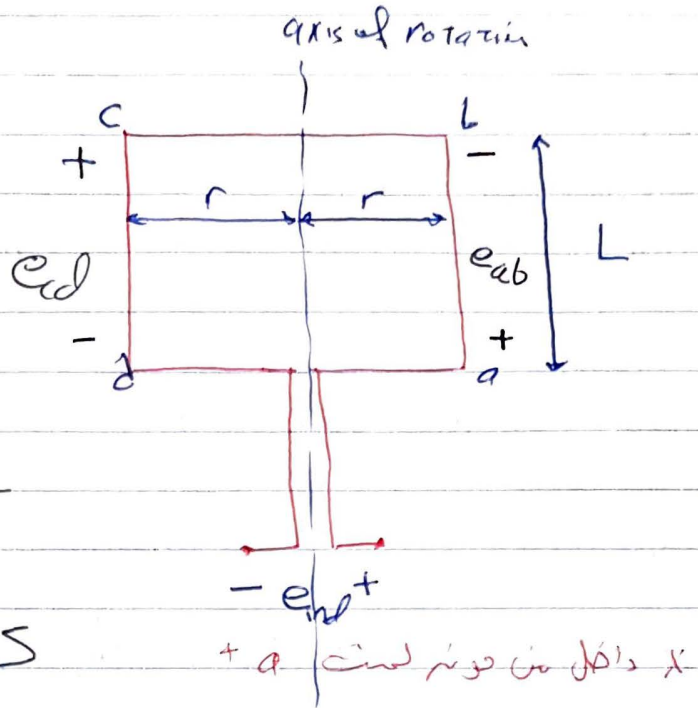
- ① Stator :- it's the stationary part and it carry high current.
- ② Rotor :- it's the rotating part in the machine.



⊗ Voltage induced in a simple Loop rotating in a uniform magnetic field

$$\text{Induced Voltage} = (\vec{v} \times \vec{B}) \cdot \vec{L}$$

- v : Velocity of conductor
- B : Magnetic flux density
- L : Length of conductor

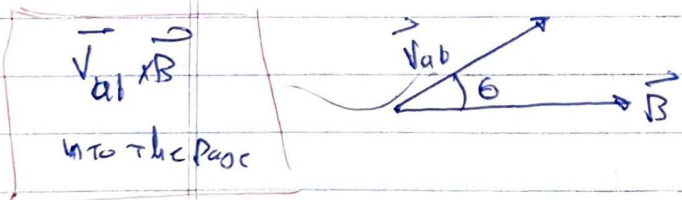


segment ab

$$e_{ab} = (\vec{v}_{ab} \times \vec{B}) \cdot \vec{L}_{ab}$$

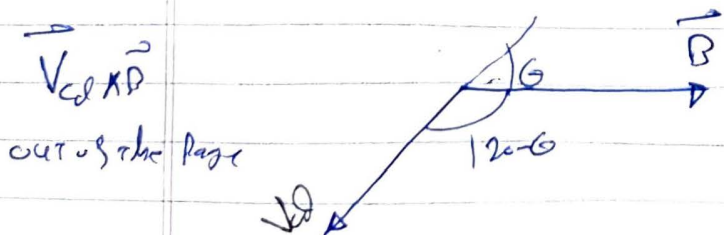
$$e_{ab} = (v B \sin \theta) L \cos(\theta)$$

$$e_{ab} = v B L \sin \theta$$



segment cd

$$e_{cd} = (v_{cd} \times B) \cdot L_{cd}, \quad L_{cd} = L$$



(ان الزاوية بين v و B هي $120 - \theta$)

$$e_{cd} = (v B \sin(120 - \theta)) L \cos(\theta)$$

$$e_{cd} = v B L \sin \theta$$

Note

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

* segment bc

$$e_{bc} = (\vec{V}_{bc} \times \vec{B}) \cdot L, \quad (\vec{V}_{bc} \times \vec{B}) \perp L(2r) \Rightarrow \boxed{e_{bc} = 0}$$

* segment ad

$$e_{da} = (\vec{V}_{da} \times \vec{B}) \cdot L, \quad (\vec{V}_{da} \times \vec{B}) \perp L(d_a) \Rightarrow \boxed{e_{da} = 0}$$

⊗ The total induced voltage is given by "KVL":

$$e_{ind} = e_{ab} + e_{bc} + e_{cd} + e_{da} = e_{ab} + e_{cd}$$

$$e_{ind} = 2VBL \sin \theta$$

$$e_{ind} = 2(\omega r)BL \sin \theta = (2rL)(B \omega_m \sin(\omega_m t))$$

$$e_{ind} = AB \omega_m \sin(\omega_m t)$$

$$\omega = \frac{d\theta}{dt} \quad \text{⊙}$$

$$\Rightarrow \theta = \omega_m t$$

where ω $A = 2rL \Rightarrow$ Area

$$e_{ind} = \phi \omega_m \sin(\omega_m t), \quad \phi = BA \text{ (flux)}$$

⊗ For N coil the induced voltage is given by ω $e_{ind} = N \phi \omega_m \sin(\omega_m t)$

In general, the induced voltage is ω

$$e_{ind} = K \phi \omega_m \sin(\theta)$$

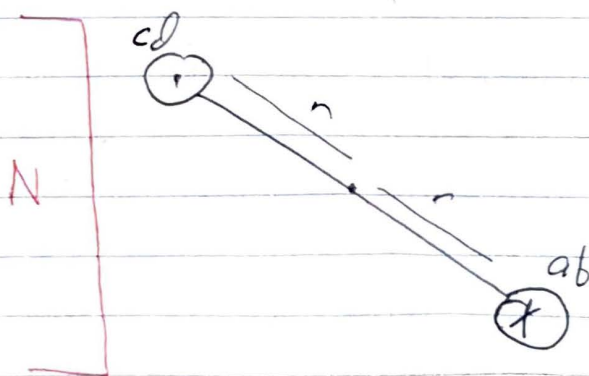
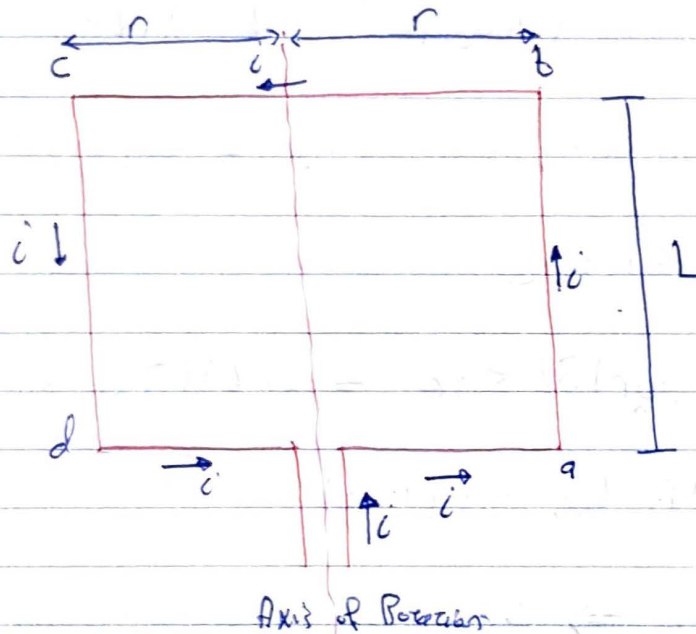
The induced voltage depend on:

- 1) Magnetic flux of the machine
- 2) speed of machine
- 3) constant represents the construction of machine (Number of turns and its shape).

⊗ Torque induced in current carrying loop in a uniform magnetic field

$$\text{Induced force} \rightarrow \vec{F}_{ind} = I \vec{L} \times \vec{B}$$

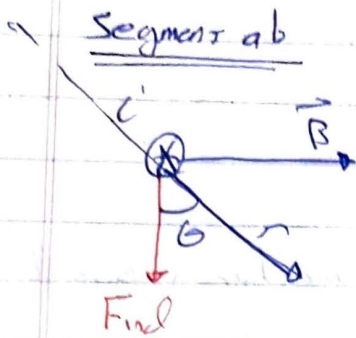
$$\text{Induced Torque} \rightarrow \vec{T}_{ind} = \vec{r} \times \vec{F}_{ind}$$



⊗ means that the current is flowing into the page

⊙ means that the current is flowing out of the page

(B) (uniform)

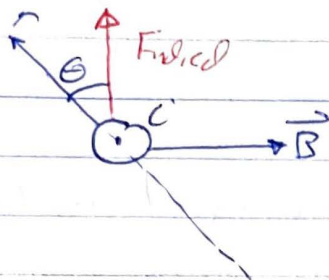


$$F_{ind,ab} = \vec{L}_{ab} \times \vec{B} = \vec{L}B \sin 90 = \vec{L}B$$

$$T_{ind,ab} = \vec{r} \times \vec{F}_{ind,ab} = r F_{ind,ab} \sin \theta$$

$$T_{ind,ab} = r \vec{L}B \sin(\theta) \text{ "clockwise"}$$

Segment cd



$$F_{ind,cd} = \vec{L}_{cd} \times \vec{B} = \vec{L}B \sin(90) = \vec{L}B$$

$$T_{ind,cd} = \vec{r} \times \vec{F}_{ind,cd} = r F_{ind,cd} \sin \theta$$

$$T_{ind,cd} = r \vec{L}B \sin \theta$$

Segment bc

$$F_{ind,bc} = \vec{L}_{bc} \times \vec{B} =$$

$$\vec{r} \text{ is parallel } F_{ind,bc} \Rightarrow T_{ind,bc} = 0$$

$$T_{ind} = \vec{r} \times \vec{F}_{ind,bc}$$

Segment da

$$\vec{F}_{ind,da} = \vec{L}_{da} \times \vec{B}, \vec{r} \text{ is parallel with } F_{ind,da}, T_{ind,da} = 0$$

⊕ The total induced torque is

$$T_{ind} = T_{ab} + T_{bc} + T_{cd} + T_{da}$$

$A = rL$
- Area.

$$= 2r \vec{L}B \sin \theta = (2rL) B \sin \theta = ABC \sin \theta$$

$$T_{ind} = \phi \dot{\theta} \sin \theta, \phi = BA$$

For n coils, the net induced torque is

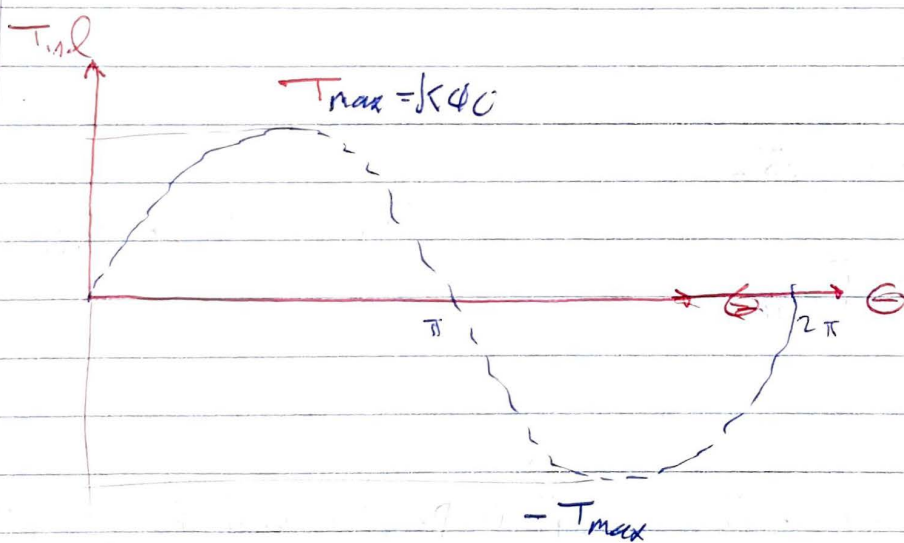
$$T_{ind} = n \phi i_s n G$$

In general, the torque induced in the machine is given by

$$T_{ind} = K \phi i_s n(\theta)$$

The torque developed by the machine depends on

- 1) magnetic flux
- 2) machine current
- 3) constant depends on the construction of the machine such as its geometry and number of turns.



and current flows in the loop, it will produce a magnetic field, B_{loop} , "Ampere's Law"

$$\oint H \cdot dl = I_{enc}$$

$$H \theta = i$$

$H = \frac{I}{G}$; G is a factor depends on geometry of the loop.

⊗ The Loop magnetic flux density $B_{loop} = \mu H = \frac{\mu I}{G}$

$$B_{loop} = \frac{\mu I}{G} \quad \text{OR} \quad I = \frac{G B_{loop}}{\mu}$$

$$I = \frac{G B_r}{\mu} \quad ; \quad B_r = B_{loop}$$

↳ Rotor magnetic field.

(B) is the external or stator magnetic field ($B = B_s$)

Recalls the induced Torque equation →

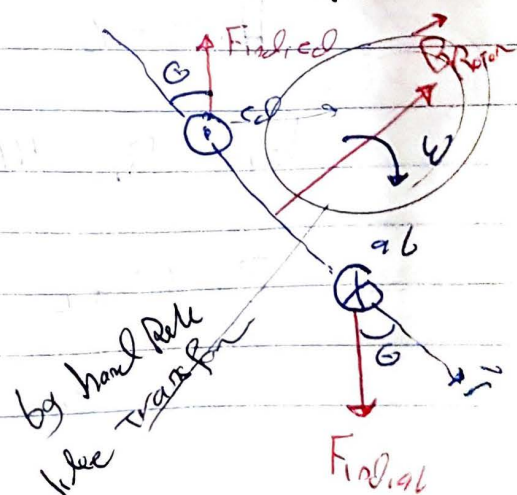
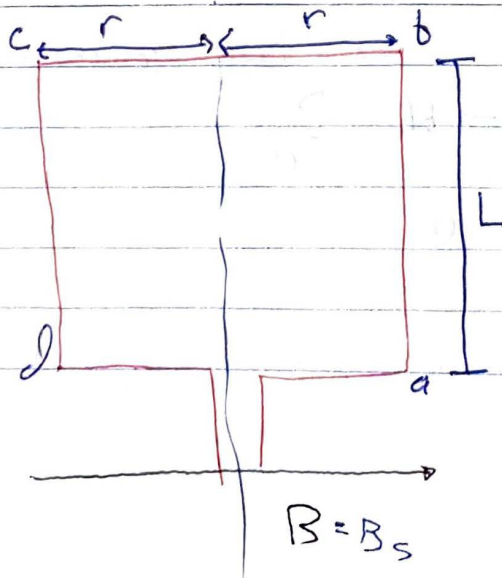
$$T_{ind} = A B_s I \sin \theta = A B_s \left(\frac{G B_r}{\mu} \right) \sin \theta$$

$$T_{ind} = \frac{A G}{\mu} B_r B_s \sin \theta$$

$$T_{ind} = K \vec{B}_r \times \vec{B}_s$$

Video

⊗ Torque induced in a simple rotating loop in a uniform magnetic field.



$$T_{ind} = 2rLB\sin\theta \quad \text{--- (1)}$$

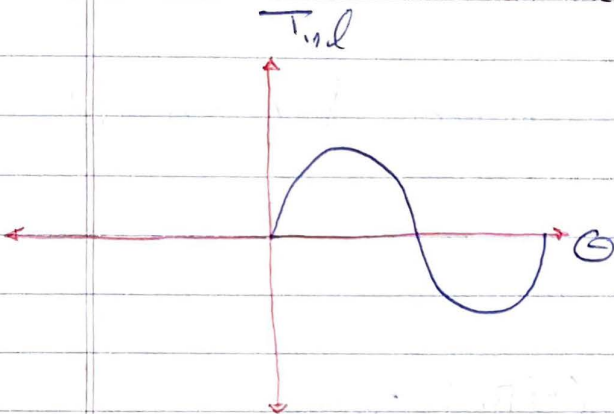
$$T_{ind} = ABi\sin\theta, \quad A = 2rL \Rightarrow T_{ind} = \phi i \sin\theta$$

⊗ For a loop, the torque developed is $T_{ind} = n\phi i \sin\theta$

In general, the induced is $T_{ind} = k\phi i \sin\theta$

Constant depends

on the construction of the machine



⊗ Alternative way to express the induced torque.

Any current flows in the loop, it will generate a magnetic field or Ampere law

$$\oint H_r \cdot dl = I_{enc} \quad , \quad \text{where } H_r \text{ is the rotor magnetic field intensity.}$$

$$H_r \theta = i$$

where θ is the factor depends on the loop geometry.

$$\text{but } B_r = \mu H_r \Rightarrow H_r = \frac{B_r}{\mu}$$

$$\frac{B_r}{\mu} \theta = i \quad \text{--- (2)}$$

by substituting equation (2) in (1)

$$T_{ind} = 2 \left(\frac{B_r G}{\mu} \right) L B_s \sin \theta$$

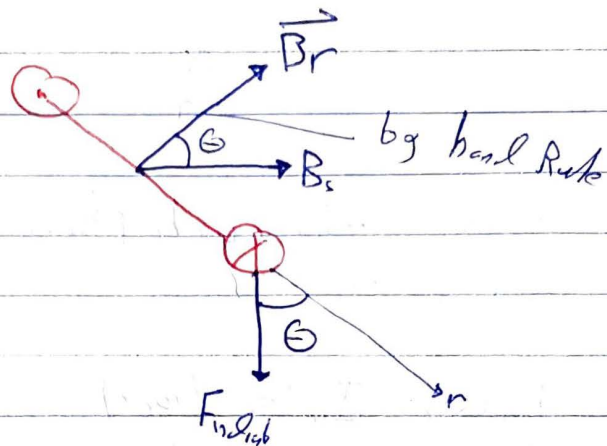
where B_s is the stator magnetic flux density.

$$T_{ind} = \left(\frac{2GL^2}{\mu} \right) B_r B_s \sin \theta$$

In general $T_{ind} = K B_r B_s \sin \theta$

The Torque induced is

$$T_{ind} = K \vec{B}_r \times \vec{B}_s$$



⊕ The Torque induced depends on:

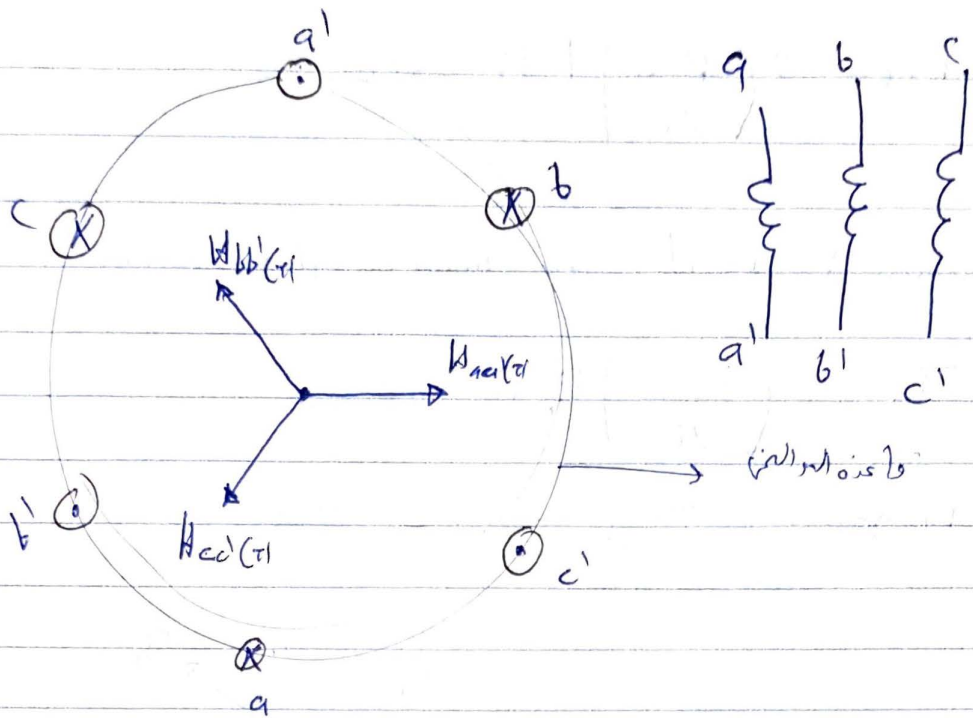
- 1) The strength of rotor magnetic field.
- 2) The strength of stator magnetic field.
- 3) Sine of angle between \vec{B}_r & \vec{B}_s .
- 4) constant representing the construction of the machine (# of turns & its shape)

Notes: \vec{B}_r & \vec{B}_s tend to align (line up) themselves with each other.

⊕ Rotating Magnetic Field is

⊕ if \vec{B}_s is rotating, the Rotor will chase it and it rotates

⊕ Assume that, we have 3-phase winding spaced electrically from each other by 120°

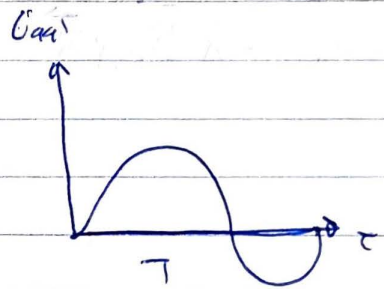


⊗ Assuming that the phases currents applied to the stator winding are

$$i_{aa'}(t) = I_m \sin(\omega_e t)$$

$$i_{bb'}(t) = I_m \sin(\omega_e t - 120^\circ)$$

$$i_{cc'}(t) = I_m \sin(\omega_e t + 120^\circ)$$



$$P_e = \frac{1}{T} \Rightarrow \omega_e = 2\pi f_e$$

⊗ where ω_e is the electric radian frequency
The magnetic field intensities are.

$$\vec{H}_{aa'}(t) = H_m \sin(\omega_e t) \underline{120^\circ}$$

$$\vec{H}_{bb'}(t) = H_m \sin(\omega_e t - 120^\circ) \underline{120^\circ}$$

$$\vec{H}_{cc'}(t) = H_m \sin(\omega_e t + 120^\circ) \underline{-120^\circ}$$

⊗ The magnetic flux densities are,

$$B = \mu H$$

$$\vec{B}_{aa'} = B_m \sin(\omega t) \angle 0^\circ$$

$$\vec{B}_{bb'} = B_m \sin(\omega t - 120^\circ) \angle 120^\circ$$

$$\vec{B}_{cc'} = B_m \sin(\omega t + 120^\circ) \angle -120^\circ$$

The net magnetic field is \vec{B}_s

$$\vec{B}_{\text{net}} = \vec{B}_s = \vec{B}_{aa'} + \vec{B}_{bb'} + \vec{B}_{cc'}$$

At $\omega t = 0^\circ$

$$\vec{B}_{aa'} = 0 \angle 0^\circ$$

$$\vec{B}_{bb'} = -\frac{\sqrt{3}}{2} B_m \angle 120^\circ$$

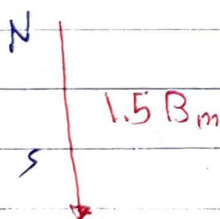
$$\vec{B}_{cc'} = \frac{\sqrt{3}}{2} B_m \angle -120^\circ$$

$$\vec{B}_s (\omega t = 0) = \vec{B}_{aa'} + \vec{B}_{cc'} + \vec{B}_{bb'}$$

$$\vec{B}_s (\omega t = 0) = \left(-\frac{\sqrt{3}}{2} B_m \angle 120^\circ + \frac{\sqrt{3}}{2} B_m \angle -120^\circ \right) B_m$$

$$\vec{B}_s (\omega t = 0) = \left[-\frac{\sqrt{3}}{2} \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) + \frac{\sqrt{3}}{2} \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right] B_m$$

$$= -j1.5 B_m = 1.5 \angle -90^\circ$$



from 3φ can we
make Rotating
Magnetic

At $\omega t = 90^\circ$

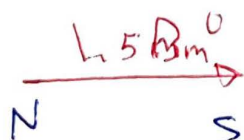
$$\vec{B}_{aa'} = B_m \angle 0^\circ$$

$$\vec{B}_{bb'} = -\frac{1}{2} B_m \angle 120^\circ$$

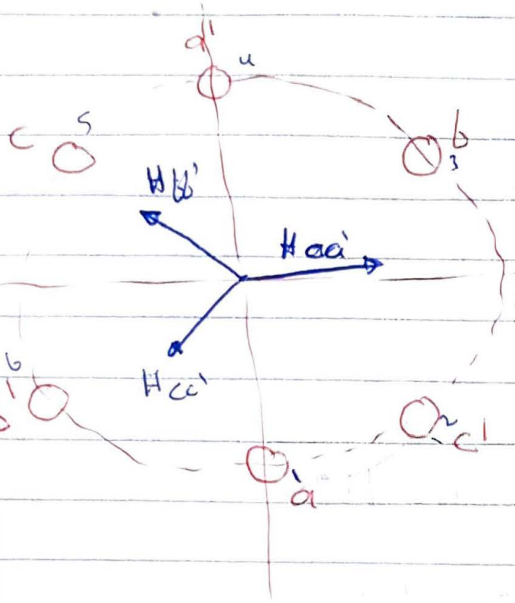
$$\vec{B}_{cc'} = \frac{1}{2} B_m \angle -120^\circ$$

$$\vec{B}_s = B_m + -\frac{1}{2} B_m \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) - \frac{1}{2} B_m \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right)$$

$$\vec{B}_s = 1.5 B_m \angle 0^\circ$$



Rotating Magnetic Field



The net magnetic field is given by

$$\vec{B}_{net} = \vec{B}_{aa'}(t) + \vec{B}_{bb'}(t) + \vec{B}_{cc'}(t)$$

when

$$B_{aa'} = B_m \sin(\omega t) \angle 0^\circ$$

$$B_{bb'} = B_m \sin(\omega t - 120^\circ) \angle -120^\circ$$

$$B_{cc'} = B_m \sin(\omega t + 120^\circ) \angle +120^\circ$$

$$\vec{B}_{net} = 1.5 B_m \sin(\omega t) \hat{x} - 1.5 B_m \cos(\omega t) \hat{y}$$

$$\omega t = 0 \Rightarrow \vec{B}_{net} = -1.5 B_m \hat{y} \quad \left| \quad \omega t = 90^\circ \Rightarrow \vec{B}_{net} = 1.5 B_m \hat{x}$$

$$\downarrow B_{net}$$

$$\rightarrow B_{net}$$

$$\omega t = 180^\circ \Rightarrow \vec{B}_{net} = 1.5 B_m \hat{y}$$

$$\uparrow B_{net}$$

⊗ The Relation between electric frequency & speed of magnetic field

⊗ winding sequence is ac'babc'b'

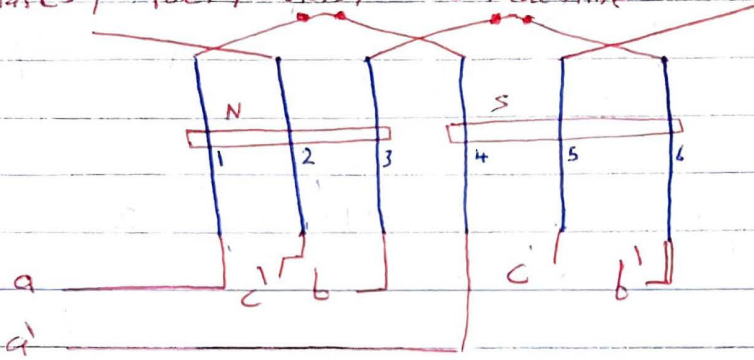
⊗ Number of poles = $P = 2$ or number of pole pair = $\frac{P}{2} = 1$

$\Theta_e = \Theta_m$ where Θ_e is electric angular position
 Θ_m is mechanical angular position

$\omega_e = \omega_m$ where ω_e = electric radian frequency ($\omega_e = 2\pi f_e$)
 ω_m = Mechanical speed (speed of rotating magnetic field)

⊗ Winding diagram of AC motor with 6 slots and 2 poles
 Rotating Magnetic Field

⊗ 3-Phases, 2 poles, 6 slots, AC machine



phase diff 120°
 slots also

$$\text{Phase difference} = \frac{120^\circ}{360} \times \frac{\text{Number of slots}}{(\text{Number of pole pairs} = P/2)}$$

$$= \frac{1}{3} \times \frac{6}{1} = 2$$

Phase - a → 1

Phase - b → 3

Phase - c → 5

⊗ if the sequence of stator winding was repeated, then we will get a 4-pole AC machine.

⊗ winding sequence = $a_1 c_1' b_1 a_1' c_1 b_1' - a_2 c_2 b_2 a_2' c_2' b_2'$

in this case, the net magnetic flux density will move half a cycle per each electric cycle.

$$\ominus m = \frac{Q_e}{2} \Rightarrow \omega_m = \frac{\omega_e}{2}$$

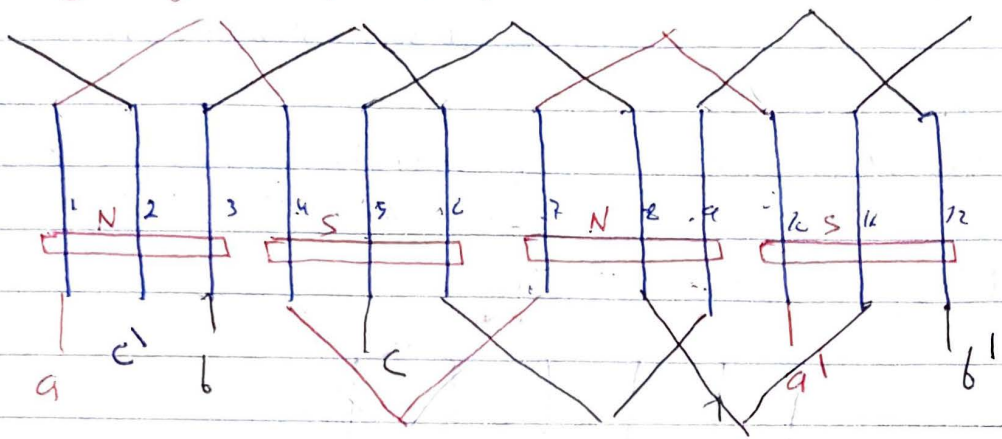
Note: B_{net} will move a complete cycle per each electric cycle

For $Q_e = Q_m$

$\omega_e = \omega_m$

external
Peaper.

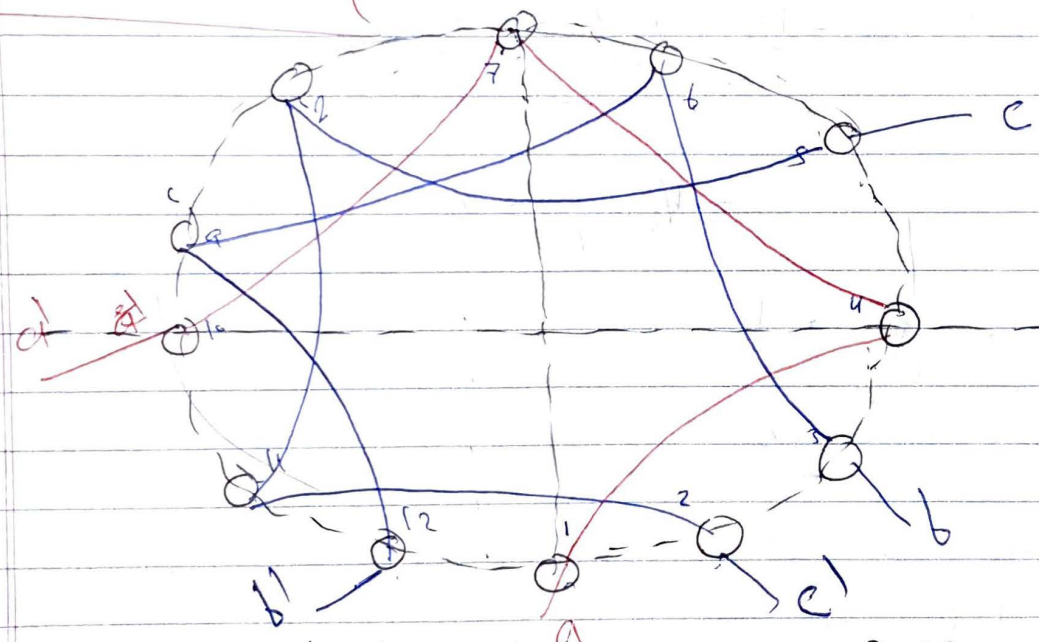
Winding diagram - 3 phase, 4 poles + 12 slots



Phase difference = $\frac{120}{360} \times \frac{\# \text{ of slots}}{\# \text{ of poles}} = \left(\frac{12}{2} \right) \left(\frac{120}{360} \right) = 2$

Phase a → 1
Phase b → 3
Phase c → 5

Ⓢ draw it from video



In general, the speed of rotating magnetic field is given by

$$\omega_m = \frac{\omega_e}{(P/2)} \quad \text{OR} \quad \omega_m = \frac{2}{P} (2\pi f_e) = \frac{4\pi f_e}{P}$$

Ⓢ The speed of rotating magnetic field in rpm, γ_m is given by

$$\gamma_m = \frac{60}{2\pi} \omega_m = \frac{30}{\pi} \left(\frac{4\pi f_e}{P} \right) \Rightarrow \gamma_m = \frac{120}{P} f_e$$

⊗ Induced Voltage in three phase - set of coils.

Assuming that the rotor is rotating at angular speed ω and it has a uniform magnetic flux density, then the magnetic flux density in the air-gap between the stator & Rotor varies sinusoidally with a mechanical angle α .

$$B = B_m \sin \alpha$$

نقطة B هي كثافة الفيض المغناطيسي في الفجوة الهوائية بين المحرك والستاتور، وتتغير جيبياً مع الزاوية الميكانيكية α .

if the number of turns in each coil is N_c , then the induced voltages at phase a, b, c are given by

$$e_{aa'} = N_c \phi \omega \sin(\omega t) \text{ Volt}$$

$$e_{bb'} = N_c \phi \omega \sin(\omega t - 120) \text{ Volt}$$

$$e_{cc'} = N_c \phi \omega \sin(\omega t + 120) \text{ Volt}$$

⊗ The Peak value of induced voltage is e_c .

$$E_{\text{peak}} = N_c \phi \omega = 2 \pi N_c \phi f$$

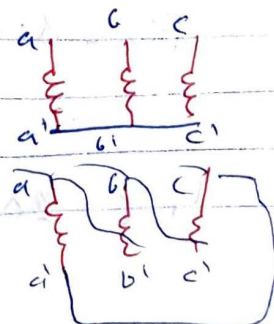
⊗ The RMS value of the induced voltage or phase rms voltage is given by

$$E_{\text{RMS}} = \frac{E_{\text{peak}}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \pi N_c \phi f$$

⊗ The line-line RMS value of induced voltage is e_c .

$$E_{L-L} = \sqrt{3} E_{\text{RMS}} \quad \text{Y-connected winding}$$

$$E_{L-L} = E_{\text{RMS}} \quad \text{Δ connected}$$



⊗ Induced Torque in Ac machine

⊗ Assume that the field distribution of stator is given by e.

$$B_{\text{stator}} = B_s \sin \alpha$$

Induced forces

$$F_{\text{ind},1} = \ell L B_s \sin \alpha$$

$$F_{\text{ind},2} = \ell L B_s \sin \alpha$$

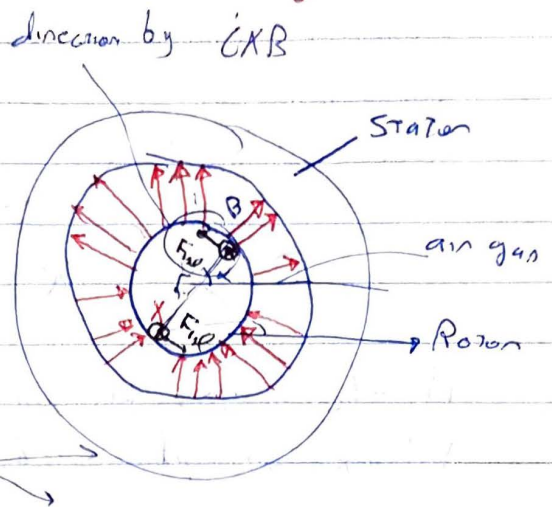
Induced Torques

$$T_{\text{ind},1} = r \ell L B_s \sin \alpha$$

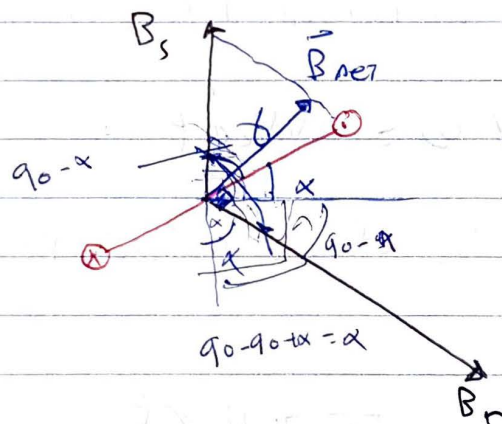
$$T_{\text{ind},2} = r \ell L B_s \sin \alpha$$

The net induced Torques

$$T_{\text{ind}} = 2r \ell L B_s \sin \alpha$$



$$\alpha = \pi - \delta$$



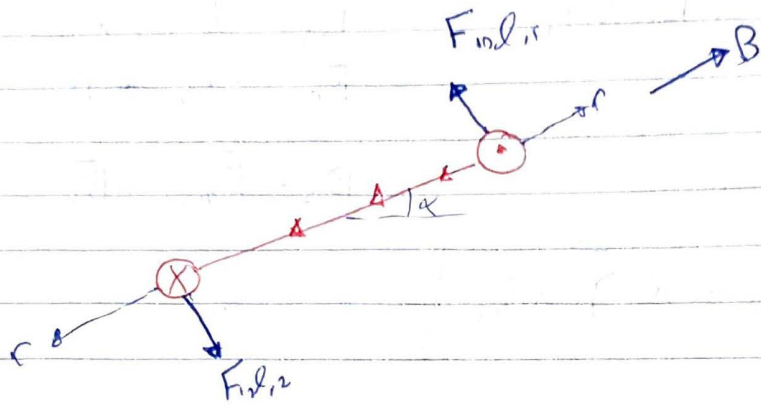
According to Ampere's law

$$\oint H_r \cdot dl = I \Rightarrow H_r = C I, \quad C \text{ is constant}$$

$$B_r = \mu H_r = \mu C I \Rightarrow \boxed{C = \frac{B_r}{\mu C}}$$

$$T_{\text{ind}} = 2r \ell L \left(\frac{B_r}{\mu C} \right) B_s \sin \alpha$$

$$= K B_r B_s \sin \alpha = K B_r B_s \sin (180 - \delta)$$



$$F_{ind,1} = l B_s \sin \alpha \Rightarrow T_{ind} = r l B_s \sin \alpha \text{ CCW}$$

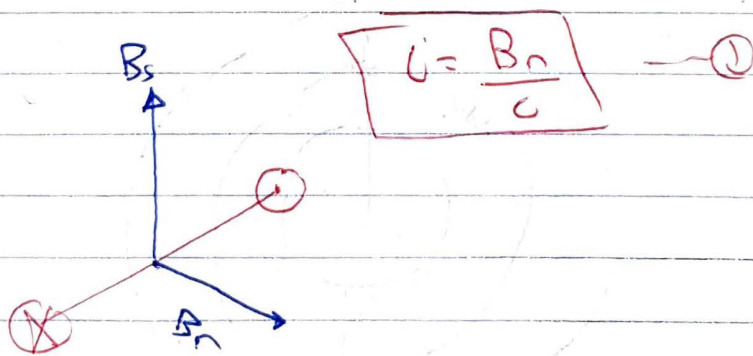
$$F_{ind,2} = l B_s \sin \alpha \Rightarrow T_{ind} = r l B_s \sin \alpha \text{ CCW}$$

⊗ The net induced torque, T_{ind} is given by

$$T_{ind} = T_{ind,1} + T_{ind,2} = 2 r l B_s \sin \alpha$$

⊗ According to Ampere's law ($\oint H \cdot dl = I_{enc} = i$), the rotor magnetic flux density, B_r , is given by

$$B_r = \mu i, \text{ where } \mu \text{ is constant.}$$

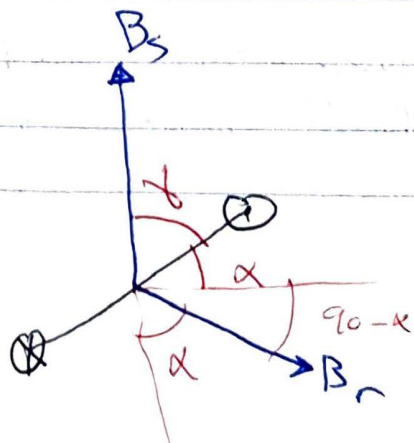


$$T_{ind} = \frac{2 r l}{c} B_r B_s \sin \alpha$$

θ_{ω} is the angle between B_s & B_r

$$\theta + \alpha = 120^\circ$$

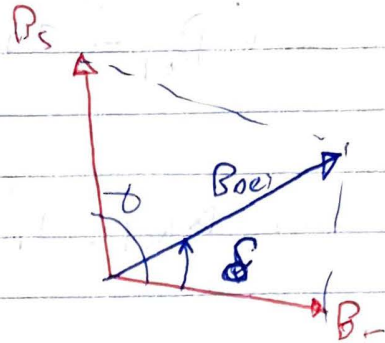
$$\alpha = 120 - \theta$$



$$T_{ind} = K B_r B_s \sin \alpha, \quad K = \frac{2pL}{c}$$

$$T_{ind} = K B_r B_s \sin(180^\circ - \delta)$$

$$T_{ind} = K \vec{B}_r \times \vec{B}_s$$



$$\vec{B}_{net} = \vec{B}_r + \vec{B}_s$$

$$\vec{B}_s = \vec{B}_{net} - \vec{B}_r$$

$$T_{ind} = K B_r \times (\vec{B}_{net} - \vec{B}_r) \Rightarrow T_{ind} = K B_r \times B_{net}$$

$$T_{ind} = K B_r B_{net} \sin \delta$$

Note if the direction of induced torque is opposite to the direction of rotation, then the machine acts as a generator.

if the direction of induced torque is the same as direction of rotation, then the machine acts as a motor.

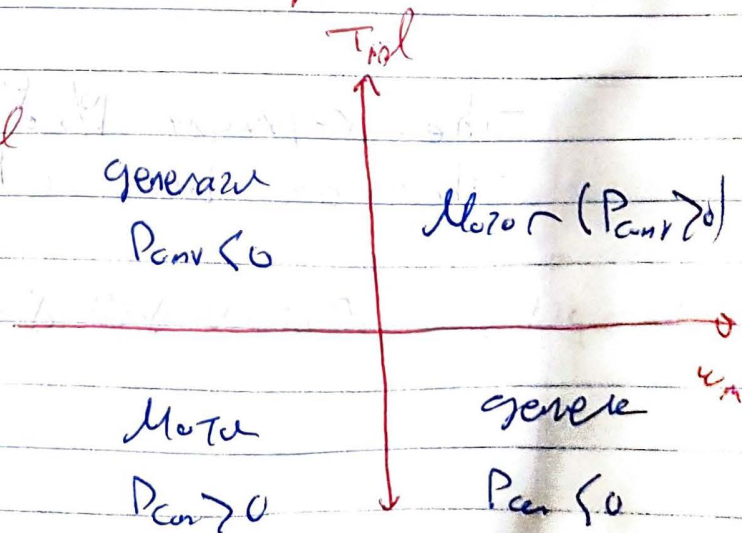
4 quadrants of operation

$$P_{conv} = T_{ind} \omega_m$$

Converted Power



Speed



⊗ Power losses in AC Machine

1) Electric or Copper losses due to the winding Resistance

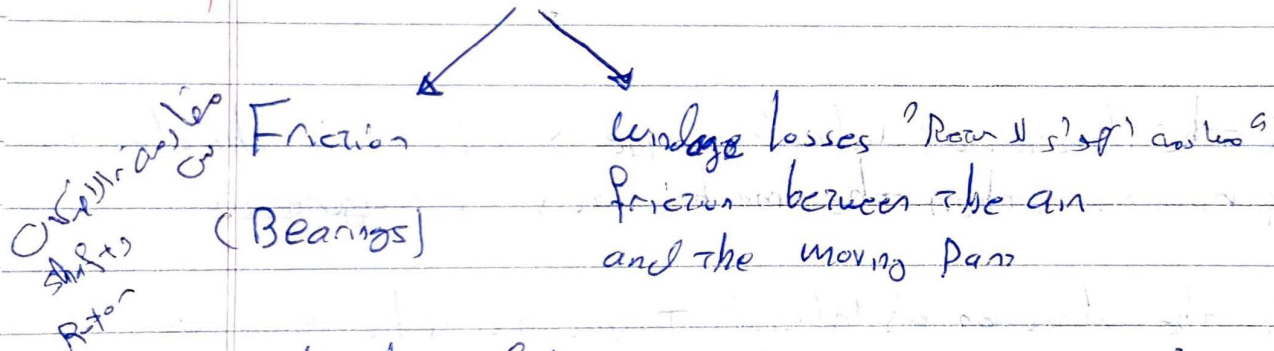
STATOR $\rightarrow P_{sel} = \sqrt{3} I_s^2 R_s$, R_s : R. STATOR Resistance
 R_r : Rotor Resistance

ROTOR $\rightarrow P_{Rr} = I_r^2 R_r$ ϕ
 Rotor copper losses

2) Core losses: eddy current & hysteresis losses

Core losses $\propto B^2 \omega_m^{3/2}$

3) Mechanical losses



Mechanical losses $\approx a_1 \omega_m + a_2 \omega_m^2 + a_3 \omega_m^3$

4) Stray losses

They represent 1% of full load

They are losses which are not accounted for

⊗ They represent 1% of full load
 They are losses which are not accounted for

Efficiency calculations of Ac Machine

The efficiency, γ of ac machine is calculated by eq

$$\gamma = \frac{P_{out}}{P_{in}} \text{ \%/}$$

For generator - operation

$$P_{in} = T_{app} \omega_m$$

$$P_{out} = 3 V_{\phi} I_{\phi} \cos \theta_{\phi} = \sqrt{3} V_L I_L \cos \theta_{\phi}$$

↑
Power factor
↖ angle between V_L & I_L

always θ_{phase}

In motor op

$$P_{in} = 3 V_{\phi} I_{\phi} \cos \theta_{\phi} = \sqrt{3} V_L I_L \cos \theta_{\phi}$$

load torque applied to the shaft

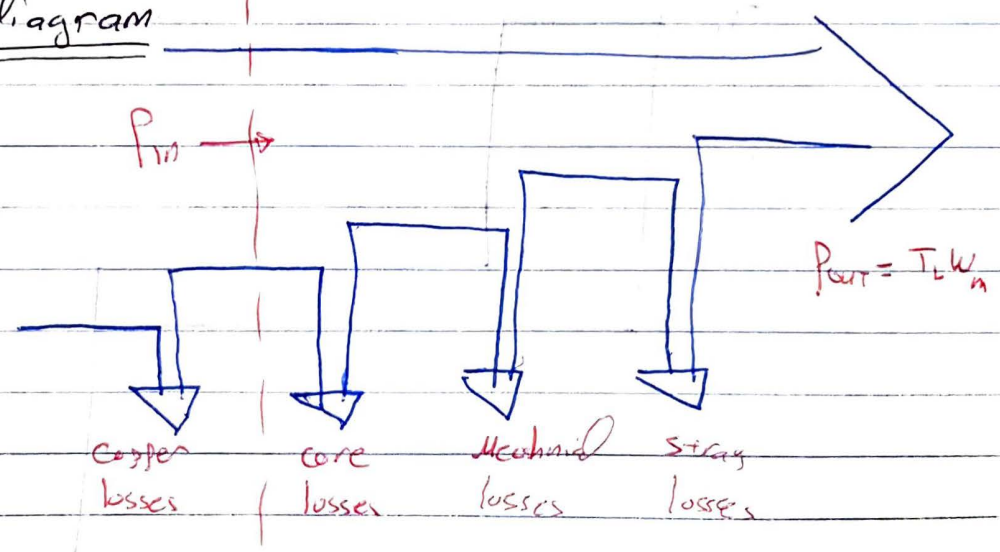
$$P_{out} = T_L \omega_m$$

$$P_{conv} = T_{ind} \omega_m$$

Power flow diagram

Motor

$$P_{in} = 3 V_{\phi} I_{\phi} PF$$



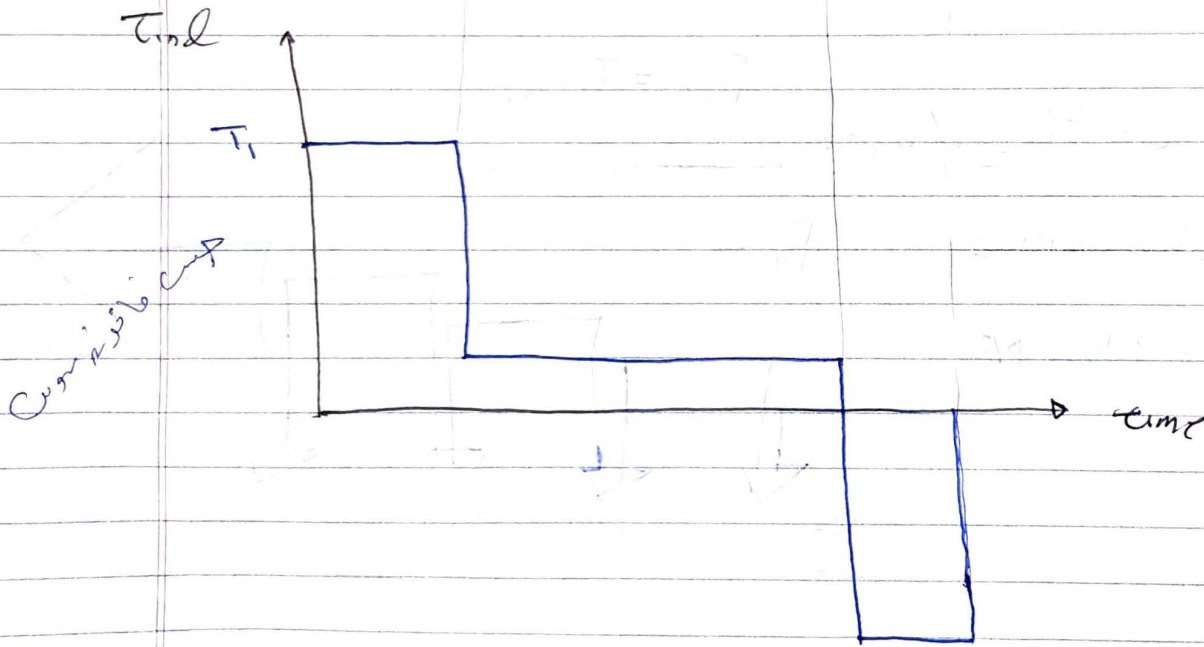
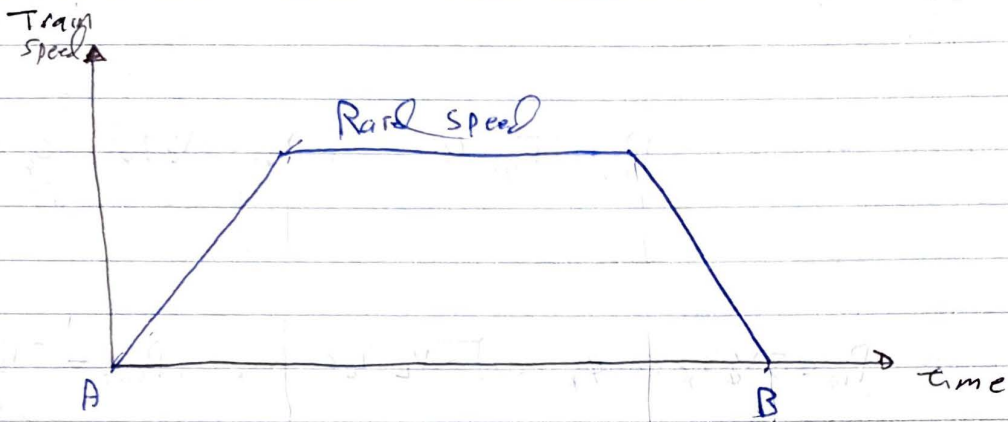
P_{conv} is the converted power to } Mechanical Motor
Electrical generator

its given by $P_{conv} = T_{ind} \omega_m$
Induced Torque

Newton's 2nd Law

$$\sum T = J \frac{d\omega}{dt} \quad ; \quad T_{ind} - T_L = J \frac{d\omega}{dt} + T_{friction}$$

Example

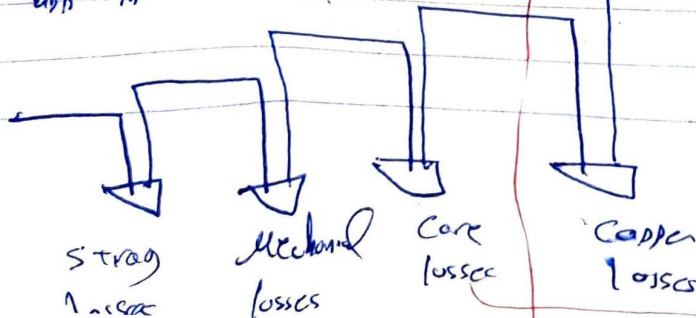


$$P_{out} = T_{ind} \omega_n$$

⊗ generator

$$P_{in} = T_{app} \omega_m$$

$$P_{out} = \sqrt{3} V_L I_L PF = 3 V_\phi I_\phi PF$$



$$P_{out} = \sqrt{3} V_L I_L PF$$

Voltage Regulation is its a measure of generation ability to keep constant voltage load condition.

$$V_R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

Speed Regulation (SR) is its ability of motor to keep constant speed under load conditions

$$SR = \frac{N_{No-load} - N_{FL}}{N_{FL}} \times 100\%$$

Full load